

USE YOUR BRAIN

THE GAME SHOW

THAT BROKE BRAINS

More Improbable Probability

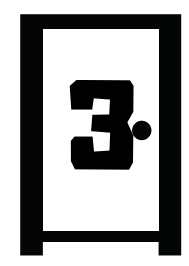
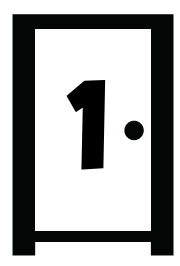


CAN YOU UNRAVEL THIS CLASSIC PARADOX?

The Problem

Today we will take a step back to the 1970's and enter the world of the classic game show "Let's Make a Deal!" You've been lucky enough to get a chance at winning the big prize, but can you figure out how to give yourself the best chance to win it? Here's the situation. . .

Monty Hall smiles at you, showing you three doors, numbered 1, 2, and 3.



You are asked to pick one, knowing that one of the options has a fabulous grand prize, like a new car, while the other two have something worthless. Unfortunately there aren't any clues so you just pick blindly. Let's say you pick #1.

After picking, Monty opens up door #2 and shows you a mangy old goat that you didn't win. He always does this, showing you one of the funny, but worthless prizes behind the door you didn't pick. He then gives you a choice: you can stay with the door you picked (#1) and get whatever is behind it or you can switch to the other closed door #3. You still have no idea what is behind either door and you have to make a decision fast.

So what do you think? If you switch will it:

- 1) Make you more likely to win
- 2) Make you less likely to win
- 3) Not change your chances of winning



The Solution?

On the actual game show, people almost always refused to switch, sticking with the door that they selected at first. They were afraid that they had picked right and by switching they would "lose" it. So did their fear help them or hurt them?

Let's break it down!

When you first pick a door you have a $1/3$ chance of picking the right one. This is because you have no information about where the prize is, so no matter which one you pick, your chance of winning is the same. In probability all of the possible options have to add up to 1. And it does!

Your chance of winning if you pick door #1: $1/3$

Your chance of winning if you pick door #2: $1/3$

Your chance of winning if you pick door #3: $1/3$

$$1/3 + 1/3 + 1/3 = 1$$



So if Monty Hall immediately opened the door you picked, you would win the big prize $1/3$ of the time.

However that isn't what happens in the game. Monty opens up one of the other doors and shows you one of the losing prizes. Now there are only two options left, so when he asks you to pick .

What most people think this means that now the chance of each door being the winning choice is $1/2$. There are two choices and we have no clues, so it makes sense that the chance of each choice is $1/2$. $1/2 + 1/2 = 1$ so that math adds up.

But we know that our original chance of winning was $1/3$. When Monty Hall opens the door, we don't get any new information about our door. Monty always shows us one of the junk prizes. It doesn't matter if our first pick was the grand prize or not.

So if we stay with our first choice, our chance of winning is only $1/3$. The other choice is to switch. We know that the sum of all of the possible options (switch or don't switch) has to add up to one and we can solve that.

$$1 - 1/3 = 2/3, \text{ so } 2/3 + 1/3 = 1$$

Our chance of winning if we switch is $2/3$! That seems unreal, but since Monty eliminates one of the junk choices, he is really letting us pick between two options. When we switch the only way we lose is if we picked the winning prize first, which was $1/3$. If we switch, we win if we picked one of the junk prizes, which is $2/3$. All of those people afraid to switch cut their chances to win in half!

If you got this wrong, don't worry. Almost everyone does at first. This isn't really a paradox because we can use math to find the solution. But the answer isn't what we expect. We think that when we pick between two unknown choices, the chances must be the same, but after careful examination we sometimes find we have more information than we think.

You'll probably never be on Let's Make a Deal to use this knowledge, but by understanding how probability can be deceiving, you might find a time in your life when you can turn the odds in your favor!