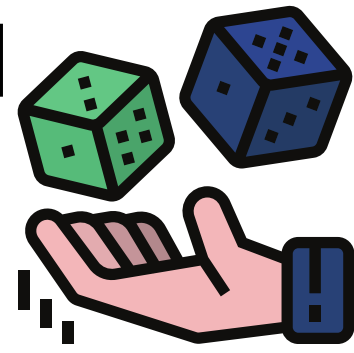


USE YOUR BRAIN

A MATHEMATIC CONUNDRUM



Improbable Probability

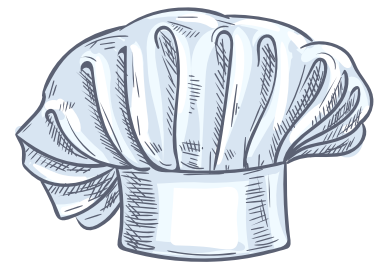
CAN YOU WRAP YOUR HEAD AROUND THIS UNEXPECTED OUTCOME?

The Problem

There are a number of versions and variations of this problem that many people see as a paradox, but all of them can be resolved the same way. Here is one version of the story. . .



A hard working Chef entered a contest and for the first time in her life, she actually won it! An all expenses paid trip for two to the Bahamas! She was so excited that she practically danced on the ceiling. Her coworkers came into the room to ask her what all the fuss was about and there was no point in lying, so she told them. Everyone was excited, but there was a problem. It was a trip for two and their were four of them! The Chef was of course going to go, she won the contest, but she had to decide who to bring: the Waiter, the Host, or the Bartender. She wasn't sure what to do, so she went home and thought about it.



She decided that picking was a bad idea (People would get mad), so she wrote down the three names, put them into a hat, drew one out and had her choice. Just than the phone rang! When the Chef answered she heard the Waiter on the line. He asked her if he would be going on the trip with her. The Chef said it was a secret and she would tell everyone tomorrow at work. The Waiter pleaded though and thought up a compromise: he asked her to tell him which of the other two people wouldn't be going on the trip. The chef didn't like the idea, but the Waiter was being pushy so she gave in told him the Bartender wasn't going. The Waiter laughed! "Now that one of them is gone, my chances are much better! 50/50!" he declared. "Thanks a lot." The Chef was very confused. Had she just given the waiter an advantage?



The Solution?

It seems obvious, right? There are only two choices left, the Waiter or the Host, so that means there is a 50/50 chance of either, right?

No. The waiter still has a $1/3$ of winning.

When the chef put three names in the hat and pulled one out, we know for sure that everyone had the exact same $1/3$ chance to be picked. The only thing that changed is that the Chef told the Waiter which of the other players didn't win. So let's examine the possibilities:

- 1) The Chef picked the Host. In this case the Chef has to tell the Waiter the Bartender isn't going.
- 2) The Chef picked the Bartender. In this case the Chef has to tell the Waiter the Host isn't going.
- 3) The Chef picked the Waiter. In this case the Chef can tell the Waiter either name.

When the Chef tells the Waiter that the Bartender isn't going, this could be case 1 or case 3. While that seems like 50/50, the Chef is only telling the Waiter that the Bartender isn't going when the Waiter gets the trip **HALF THE TIME**. The other half of the time she will be saying the Host. Meanwhile when the Host gets picked for the trip, the chef **ALWAYS** tells the Waiter that the Bartender won't be going. Something that **ALWAYS** happens is twice as likely as something that happens **HALF THE TIME**, so the odds that the waiter is on the trip is still $1/3$, half of the $2/3$ chance he isn't going.

As we know the chance of the bartender going is 0, this means that the Host's chance of being picked are $2/3$. So did the Host's chances of winning get better?

Sort of. Everyone's chance was always $1/3$ before the chef picked the name. After she picked the name, the winner is already picked. However only one person knows this (the chef). Without any other information, everyone else can only guess based on the odds, but in this case there is additional information. The reason the Waiter's chances didn't change is because the Chef didn't add any information about whether or not the Waiter won!. But she did add some information about whether or not the Bartender and the Host won. Now that we know the Bartender didn't win, the Bartender's $1/3$ chance of winning has to go somewhere (there is no chance that no one won). And since we didn't learn anything about the Waiter there is only one place it could have went: the Host. The Waiter wanted to learn about his chances, but he only learned about the chances of the other two!

Were you able to understand that? It's a tough one. Next we do more probability and try to unravel one of the classic paradoxes from game show history!