While you are out for a walk with your friend an honest stranger makes you an offer. You have a choice two envelopes. The stranger explains that one envelope has twice as much money as the other. You make your pick and go to open it when the stranger tells you to wait. The stranger asks you if you'd like to switch envelopes. You laugh and start to say no, but your friend tells you to wait.

"Imagine this," your friend says. "Let's say the envelope your holding has $20 in it. That means the other one could have twice as much! \( 2 \times 20 = 40 \)!

"But what if this envelope is the one with twice as much?" you ask.

"I thought about that," your friend explains. "That would mean the other envelope has $10 because \( 2 \times 10 = 20 \). So do the math. If you're right, you'll get $20 more, because \( 40 - 20 = 20 \). If you're wrong, you'll only get $10 less, because \( 20 - 10 = 10 \). You should definitely switch!"

Do you think your friend is right, is it just nonsense?
Your friend's math seems very convincing, but sadly your decision doesn't change your chances. When you chose an envelope you didn't have any information and no one gave you any new information, so switching can't change anything as its the same as choosing a different envelope in the first place. On top of that, if you switch twice, you'd go back to having the same envelope which means you'd have the same amount of money, which doesn't make sense if switching makes you money. Further, if two people have envelopes and they switch with each other it can't be possible that one of them gets $20 and one loses $10. Switching envelopes clearly doesn't make money appear in the envelopes.

So what's wrong with your friends math? The problem is your friend is comparing two different possibilities, but in reality there are only one of them is true. In the first situation, there is a total of $20+$40=$60 in both envelopes. In the second situation there is $10+$20=$30 in both envelopes. There is twice the amount of money in the first situation! So your friend is right that in the first situation you get more money by switching, but you also get more money by not switching! No matter what you do, you get twice as much money as you would have if in the second situation because both envelopes have twice as much money in them. Switching or not switching doesn't matter. The critical difference happened before you arrived.

The Solution

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